Virtual Transitions to the Continuum in O^{16}

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ERRATA

Page 2 Eq. (5) Second term on the right E_μ should be replaced by E

Page 3 Eq. (7) should read

$$\Gamma_\mu(E') = 2\pi \int \left| \langle \chi_\mu \,|\, \tau_e \,|\, \chi^{(+)}(E') \rangle \right|^2 \rho_c\,(E')\,dc$$

where ρ_c (E') is the density of continuum states at energy E', and c refers to the quantum numbers and solid angle of the continuum particle.

Page 3 Eighth line up should read . . . .

in τ_h. The corrections to the energy shift due to the off diagonal matrix elements of τ_e between the states.

Virtual Transitions to the Continuum in O^{16}†

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In a shell model calculation, one mixes the simplest configurations by a two-body interaction, thus obtaining energy levels. The wave functions are then used to calculate gamma transition probabilities and cross sections for various nuclear reactions. The interaction strength and exchange mixture parameters are adjusted to agree with some experimental observation, e.g., the level energies. In certain instances it is necessary to introduce admixtures of more complex configurations. The effect of the high momentum components of the repulsive hard core are usually included in the energy independent two-body interaction. Seldom considered, however, are the virtual transitions to low lying continuum states. As a test case, the shifts of the negative parity levels of O^{16} due to these low momentum off the energy shell transitions of both neutrons and protons have been calculated. Their contributions are found to be significantly large.

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The shifts are calculated using the theory of MacDonald[1]). In this scheme the effective nuclear interaction τ_e is given by

$$\tau_e = v + v(E^+ - H_o)^{-1}\, \Pi_c\, \tau_e \qquad (1)$$

where v is the residual interaction, i.e., the difference between the actual and model Hamiltonians H and H_o. The operator Π_c projects onto the continuum states of H_o. Garside[2]) has suggested writing

$$\Pi_c = \Pi_h + \Pi_\ell \qquad (2)$$

where Π_h and Π_ℓ project respectively onto the high and low momentum continuum states of H_o. He then defines an effective interaction τ_h as

$$\tau_h = v + v(E^+ - H_o)^{-1}\, \Pi_h\, \tau_h \,. \qquad (3)$$

Using eqs. (1), (2), and (3) then yields

$$\tau_e = \tau_h + \tau_h(E^+ - H_o)^{-1}\, \Pi_\ell\, \tau_e \,. \qquad (4)$$

Because τ_h includes all the high momentum components of the two nucleon correlations it can be chosen as energy independent and without a hard core. The second term of eq. (4) considers explicitly low momentum correlations. If the μ'th state of $H_o + \tau_h$ is represented by X_μ then the diagonal matrix elements are

$$\langle X_\mu | H_o + \tau_e | X_\mu \rangle = \langle X_\mu | H_o + \tau_h | X_\mu \rangle + \frac{1}{2\pi}\; P \int_0^\lambda \frac{\Gamma_\mu(E')dE'}{E_\mu - E'} \,, \qquad (5)$$

or symbolically

$$\mathcal{E}_\mu(E) = E_\mu + \Delta_\mu(E). \qquad (6)$$

The eigenvalue E_μ of X_μ is given by the first term on the right hand side of eq. (5). The diagonal shift due to virtual transitions to the low momentum continuum $\Delta_\mu(E)$ is given by the second term on the right in eq. (5). In this term P denotes the principal value of the integral and $\Gamma_\mu(E')$ is the width function of the level μ given by

$$\Gamma_\mu(E') = 2\pi |\langle X_\mu | \tau_e | \chi^{(+)}(E')\rangle|^2 , \qquad (7)$$

with $\chi^{(+)}(E')$ the continuum eigenfunction of H_o. If $\mathcal{E}_\mu(E_{Res}) = E_{Res}$ is the resultant energy of the μ'th compound level, then $\Gamma_\mu(E_{Res})$ is just the usual definition of the decay width. The upper energy limit λ in the integral of eq. (5) is arbitrarily chosen as 65 MeV which corresponds to a characteristic particle separation of about .5 f. Energies above this are associated with short range correlations generated by the repulsive core and are included in τ_h. The off diagonal elements of τ_e between the states X_μ and X_η are of second order in Im τ_e << Re τ_e and via the optical theorem of fourth order in τ_e [3]. These have been calculated by Garside[2] and are only of the order of 0 - 40 keV. Consequently these terms are neglected.

In the present calculation the states X_μ are obtained in a particle-hole basis of states consisting of $p^{-1}_{1/2}\, s_{1/2}$, $p^{-1}_{1/2}\, d_{3/2}$, $p^{-1}_{1/2}\, d_{5/2}$, $p^{-1}_{3/2}\, s_{1/2}$, $p^{-1}_{3/2}\, d_{3/2}$, and $p^{-1}_{3/2}\, d_{5/2}$.

The interaction τ_h is chosen to have two parts, viz:

$$\tau_h = \tau_{h(P.h.)} + \tau_{h(1d_{3/2})} \tag{8}$$

The first term is the particle-hole interaction while the second is a one-body potential which exists only for the $1d_{3/2}$ state. This state is a single-particle resonance in O^{17} and F^{17} and in ref. 4) the treatment of this level as an intermediate bound state which is pushed up into the continuum by $\tau_{h(1d_{3/2})}$ has been described. This removes the sharp energy dependence from the expression for $\Delta_\mu(E)$. Following ref. 4) $\tau_{h(1d_{3/2})}$ is just the extra well depth needed to bind the $1d_{3/2}$ state. Because it is easy to calculate, the spin dependent particle-hole residual interaction with delta function radial dependence used by Brown, et. al.[5]) and Lemmer and Shakin[6]) viz.:

$$\tau_{h(P.h.)} = - V_o\ [.865 + .135\ \vec{\sigma}_1 \cdot \vec{\sigma}_2]\ \delta(\vec{r}_1 - \vec{r}_2) \tag{9}$$

is employed. The strength V_o is that used in these references and is 581 MeV f^3.

The width functions eq. (7) and energy shifts Δ_μ have been calculated using the code ABACUS[7]) with neutron and proton wave functions obtained from a Woods-Saxon well with spin-orbit coupling and coulomb potential of a uniform sphere of charge. The parameters for this and $\tau_{h(1d_{3/2})}$ are discussed in ref. 8) where the energies E_μ and width of the 1^- states of O^{16} have been calculated. In evaluating $\Gamma_\mu(E')$, the effective interaction τ_e has been approximated by τ_h.

In fig. 1 are plotted the neutron and proton width functions for a typical level (the J = 1^-, 22.6 MeV level of ref. 8)). The shapes are due to the energy dependence of the $s_{1/2}$, $d_{3/2}$, and $d_{5/2}$ single particle continuum wave functions. The opening of the $p^{-1}_{1/2}$ neutron (proton) channel at 15.67(12.13) MeV makes the width zero at energies below these. The bumps near 20 MeV are caused by the opening of the $p^{-1}_{3/2}$ neutron and proton channels at 21.83 and 18.45 MeV respectively.

Figure 2 presents for the five J = 1^- levels a plot of $\mathcal{E}_\mu(E) - E$ vs. E. The intersection with the zero ordinate line gives the resonant energies. The curves are seen to be very nearly linear, and none of the possible multi-intersections suggested in ref. 6) occur. Similar curves are obtained for the J = 0^-, 2^-, 3^-, and 4^- levels. It is noted that the resultant energies $\mathcal{E}_\mu(E_{Res})$ are considerably below the experimental energies (c.f. table 1), e.g., the 21.3 MeV intersection in fig. (2) is associated with the experimental 22.4 MeV state.

In table 1 are listed for J^π = 0^-, 1^-, 2^-, 3^-, and 4^-, the energies E_μ prior to the continuum shift, the known experimental energies, the total neutron and proton shifts and partial shifts for the $p^{-1}_{1/2}$ and $p^{-1}_{3/2}$ channels, and the resultant compound nuclear energies $\mathcal{E}_\mu(E_{Res})$. All shifts are at $\mathcal{E}_\mu(E_{Res}) - E_{Res} = 0$. Also included are the neutron, proton and total widths at the energies E_μ. The shifts are negative because there are more continuum states above than below E_μ in the integral eq. (5). They vary from -.08 to -1.7 MeV and are in many cases of the order of the level widths (table 1). That these two effects are comparable has previously been mentioned by Ferrell[9]). The large level shifts result in several instances of serious disagreement with experiment. This is most evident for the 1^- levels about which more is known. In order to correct for the large shifts a greater particle-hole force strength must be used. Buck and Hill[10]) have predicted the photonuclear resonance cross sections in O^{16} using a coupled channel calculation which automatically includes coupling to the continuum. They find that the best agreement with experiment comes from using a force strength of 650 MeV f^3, somewhat greater than the value used in refs. 5), 6), and the present case. A very quick calculation of this effect may be made for the $J = 1^-$ levels. These states are predominantly of one particular particle-hole configuration[5,8]), and the change in energies E_μ due to the additional force strength is given very nearly by the change in the diagonal element of the dominant configuration. This raises the values of $\mathcal{E}_\mu(E_{Res})$ to 23.7, 21.9, 20.0, 17.1, and 13.7 MeV which are in better agreement with their experimental counterparts (table 1). The widths in ref. 10) are somewhat larger than in the present calculation because of the stronger interaction used.

The indication is that virtual transitions to the continuum are important and when properly taken into account require a greater phenomenological two body interaction than that required in the usual shell model calculations.

Lastly it should be mentioned that a calculation of the low momentum correlations using more realistic two-body interactions (e.g., the effective interaction derived by Kuo and Brown[11]) would be very useful.

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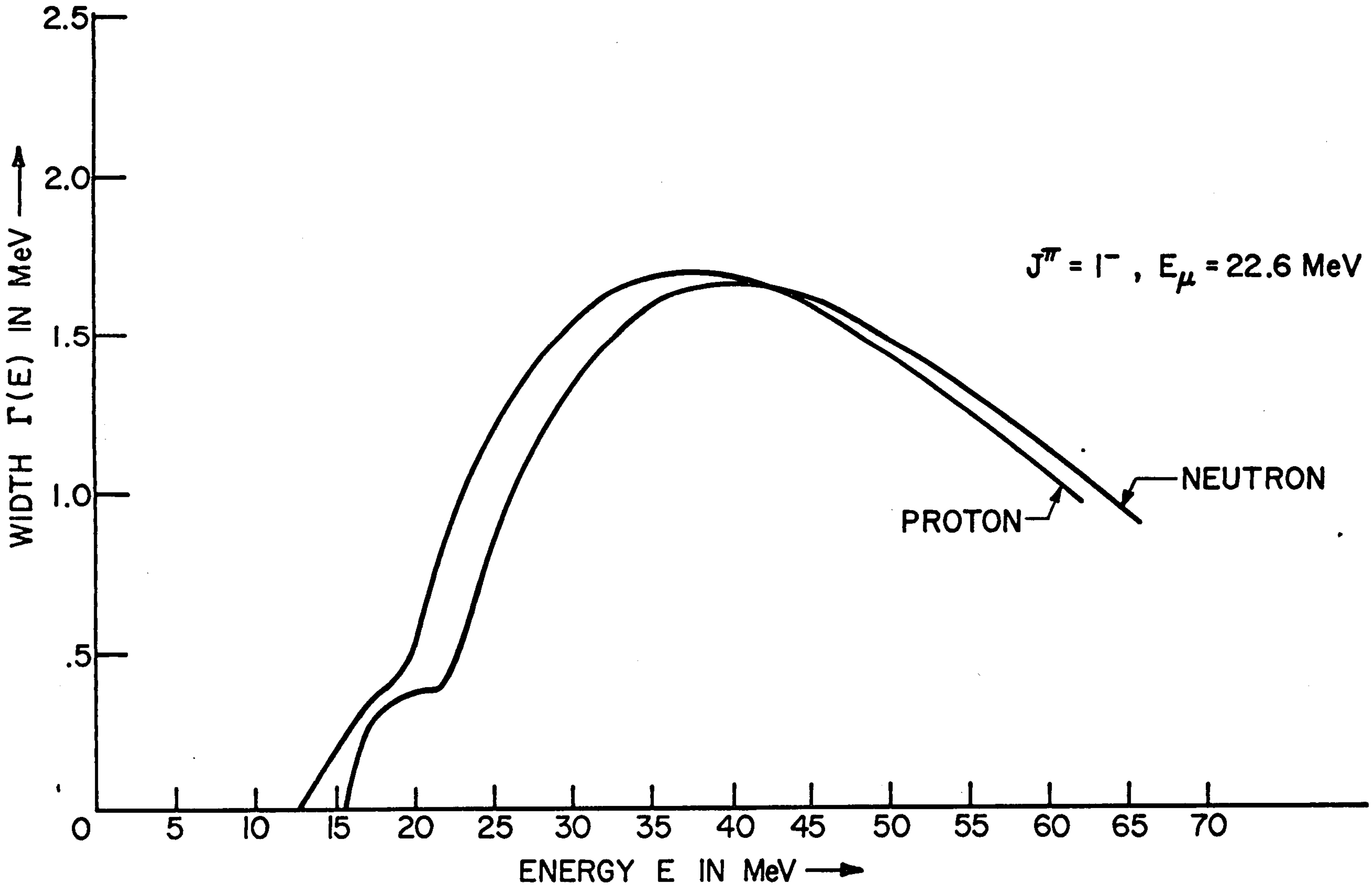
Figure Captions

Fig. 1 Neutron and proton width functions in O^{16} vs. energy for a typical level (the $J = 1^-$, 22.6 MeV level of ref. 9).

Fig. 2 Diagonal matrix element $\mathcal{E}_\mu(E)$ minus E vs. E for the five $J = 1^-$ states. The intersections with the zero ordinate line yield the resultant compound nucleus energies $\mathcal{E}_\mu(E_{Res})$.



$J^{\pi} = 1^{-}$, $E_{\mu} = 22.6$ MeV
NEUTRON
PROTON
2.5
2.0
1.5
1.0
.5
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
WIDTH $\Gamma(E)$ IN MeV
ENERGY E IN MeV

FIG. 1

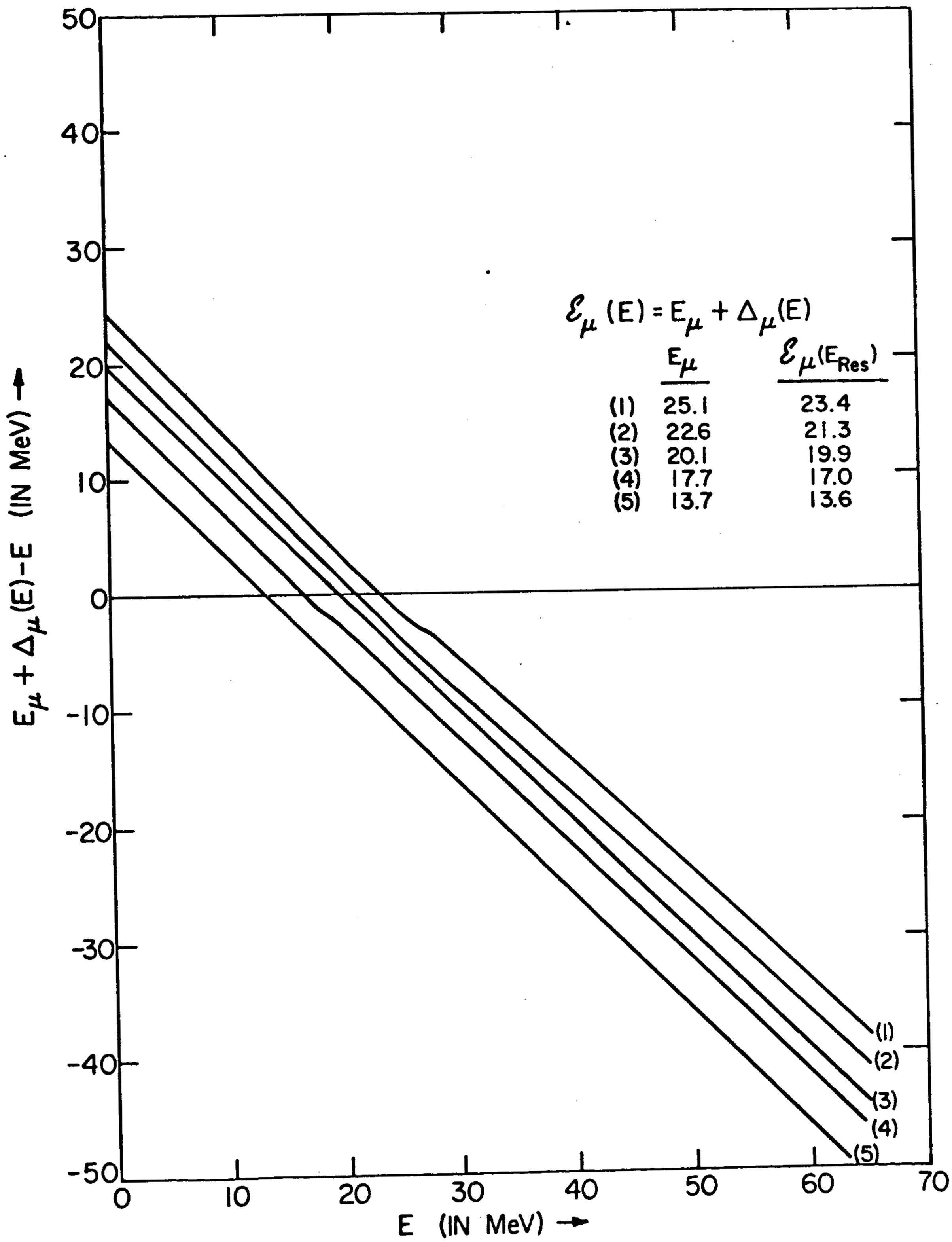

$\mathcal{E}_\mu(E) = E_\mu + \Delta_\mu(E)$
E_μ
$\mathcal{E}_\mu(E_{Res})$
(1) 25.1 23.4
(2) 22.6 21.3
(3) 20.1 19.9
(4) 17.7 17.0
(5) 13.7 13.6
$E_\mu + \Delta_\mu(E) - E$ (IN MeV) →
E (IN MeV) →
50
40
30
20
10
0
-10
-20
-30
-40
-50
0
10
20
30
40
50
60
70
(1)
(2)
(3)
(4)
(5)

FIG. 2

Table 1

Energy Shifts at $\mathcal{E}_\mu(E_{Res}) - E_{Res} = 0$ for the Negative Parity Levels of O^{16} Due to Virtual Transitions to the Continuum (all energies in MeV)

J^π	E_μ	E_{exp}	Neutron Shifts X(-1)			Proton Shifts X(-1)			Total Shift X(-1)		widths at energy E_μ		
			$p_{1/2}^{-1}=a$	$p_{3/2}^{-1}=b$	$\Delta_N=a+b$	$p_{1/2}^{-1}=c$	$p_{3/2}^{-1}=d$	$\Delta_p=c+d$	$\Delta_\mu=\Delta_N+\Delta_p$	$\mathcal{E}_\mu=E_\mu+\Delta_\mu$	Γ_n	Γ_p	Γ_{n+p}
0^-	25.70		.03	.35	.37	.02	.43	.45	.82	24.9	1.04	2.58	3.62
	13.46	12.78	.03	.00	.04	.05	.01	.06	.10	13.4	0	.09	.09
1^-	25.11	24.5	.11	.35	.46	.25	.94	1.2	1.7	23.4	1.02	1.96	2.98
	22.58	22.4	.17	.39	.56	.26	.49	.75	1.3	21.3	.43	.89	1.32
	20.13	(20.45)	.04	.09	.13	.05	.12	.17	.30	19.8	.01	.23	.24
	17.69	17.3	.14	.02	.16	.52	.01	.53	.69	17.0	.18	.70	.88
	13.70	13.1	.05	.01	.06	.10	.01	.10	.16	13.5	0	.12	.12
2^-	23.51		.02	.10	.13	.05	.62	.67	.80	22.7	.23	.80	1.03
	19.76		.04	.06	.10	.02	.06	.08	.18	19.6	.08	.21	.29
	19.42		.07	.04	.10	.15	.05	.20	.30	19.1	.10	.23	.33
	17.73		.14	.01	.15	.26	.00	.27	.42	17.3	.21	.76	.97
	12.75	12.96	.02	.02	.04	.03	.02	.04	.08	12.7	0	.00	.00
3^-	25.22		.00	.23	.23	.01	.61	.61	.84	24.4	.81	2.23	3.04
	18.82		.08	.02	.10	.05	.03	.08	.18	18.6	.05	.10	.15
	13.28	13.26	.05	.01	.06	.06	.01	.07	.13	13.2	0	.01	.01
4^-	19.72		--	.06	.06	--	.08	.08	.14	19.6	.00	.01	.01